In Part v. the subject of Forest Reserves is elaborately dealt with by Mr. Henry Gannet and others. An endeavour is made to estimate the present amount of woodland distributed in the different States. Texas has the largest area, of about 64,000 square miles, while Arkansas has the largest percentage of woodland. The question of the protection of forests is one that is engaging much attention, so that the statistics and general information here brought together must be of great value. The report is illustrated by an atlas.

Of the Twentieth Annual Report we have received Part i. and Part vi. (2 vols.). Part i. contains the report of the Director, Mr. Charles D. Walcott, an admirable record of systematic work, which evidently receives the sympathy and substantial support of Congress. The appendix contains details of triangulation and spirit-levelling, and the work is accompanied by maps showing the progress of the surveys. Part vi. is on the Mineral Resources for 1898, the subject being under the direction of Mr. David T. Day. The total value of the mineral productions is the largest ever recorded in the history of the United States. All the metals, except nickel, made large gains, copper, lead, zinc, aluminium and antimony reaching their maximum, both in production and value. The amount of pig-iron produced was greater than in any other year, but the value was less. The non-metallic products also show an increase, especially bituminous coal, and in a lesser degree stone, petroleum and natural gas. The coal product amounted to about two hundred millions of tons.

Monographs.

Monograph No. 32, Part ii., is a large and handsome volume on the Geology of the Yellowstone National Park, by Mr. Arnold Hague and numerous colleagues. It opens with an account, by Messrs. J. P. Iddings and W. H. Weed, of the Gallatin Mountains, which consist of sedimentary strata ranging from Cambrian to Carboniferous, Jura-Trias, and Cretaceous (Laramie). Disturbances at the close of the Laramie formation were accompanied by igneous intrusions in the form of large laccolites, mainly andesitic in character. Electric Peak and Sepulchre Mountain are described as parts of a Tertiary volcano which was faulted across the conduit, the amount of vertical displacement having been more than 5000 feet.

Mr. Hague describes a mountainous area in the southern part of the Park, comprising ridges formed partly of Palæozoic but chiefly of Cretaceous rocks. The irregular outline of the mountains is due to the rhyolites of the Park Plateau that abut against the slopes of the upturned sedimentary strata. The Snake River hot springs are situated near the contact of the rhyolite with the Madison (Carboniferous) limestone, whence the travertine of the springs is derived. Mr. Iddings gives a particular account of the Miocene volcano of Crandall Basin, which arose on a ridge of Palæozoic rocks and on remnants of Eocene breccias and lava flows. The volcano consisted of andesitic breccias capped by basalt flows and traversed by dykes. It must have risen 13,400 feet above the limestone floor. The igneous rocks of the Absaroka range, and others which lie within Yellowstone Park, are specially dealt with by Mr. Iddings. The Cambrian fossils are described by Mr. C. D. Walcott, the Devonian and Carboniferous by Mr. G. H. Girty, the Mesozoic by Mr. T. W. Stanton, and the Fossil Flora (Laramie and Tertiary) by Mr. F. H. Knowlton.

Monograph No. 33 contains an account of the geology of the Narragansett Basin, a tract which includes Providence on the north and Newport on the south, being parts of Rhode Island and Massachusetts. The section on general geology is contributed by Mr. N. S. Shaler, while the detailed accounts are furnished by Mr. J. B. Woodworth and Dr. A. F. Foerste. Mr. Shaler remarks that the region originally contained an extensively developed series of pre-Cambrian ro ks, which "may for convenience be referred to that limbo of ill-discriminated formations, the Upper Archæan (of Dana), or Algonkian." On these lie remnants of the Olenellus-beds of the Lower Cambrian, and above these are granites which have broken through the Cambrian, and have in turn been much eroded. On top lie the Carboniferous strata, which occupy the greater part of the basin and attain a thickness of several thousand feet. The general proposition is that this and other basins which lie along the Atlantic coast from Newfoundland to North Carolina are old river valleys which have been depressed below the sea-level, filled with sediments—the sedimentation increasing the depth of the depression—and afterwards corrugated by the mountain-building forces. The memoir is well illustrated with maps, sections and pictorial plates.

Monograph No. 34 is on the glacial gravels of Maine and their associated deposits, by Mr. George H. Stone. The subject is treated with a wealth of letterpress (499 pages) and illustrations. It is essentially a local memoir, but as the result of careful observations commenced so long ago as 1876, it is a most valuable record of facts on water-assorted glacial drift, useful to those studying glacial features, terraces, eskers and the probable effects of subglacial and englacial streams.

In Monograph No. 36 the Crystal Falls iron-bearing district of Michigan is described by Messrs. J. Morgan Clements and H. Ll. Smyth. This is the third of a series of reports on the iron-bearing districts of Lake Superior. The iron-ore (hæmatite and limonite) occurs in the Upper Huronian series. It is associated with white and reddish chert, and lies between carbonaceous slates in synclinal troughs. The memoir, however, deals with the structure, stratigraphy and physiography of a large area, approximately 540 square miles, and not only with Archæan and Huronian, but more particularly with various volcanic and intrusive rocks, microscopic sections of which form a main feature in the illustrations. A general introduction is written by Mr. C. R. Van Hise, and a final chapter on the Sturgeon River tongue in the south-eastern part of the district is by Mr. W. S. Bayley.

Monograph No. 37 is on the Fossil Flora of the Lower Coal-measures of Missouri, by Mr. David White, and is illustrated by seventy-two plates of Carboniferous plants, and one of a coal-seam.

Monograph No. 38, a large volume of 817 pages, numerous maps and other illustrations, is given up to a description of the Illinois Glacial Lobe, by Mr. Frank Leverett. This ice-tract formed the south-western part of the great ice-field that formerly extended from the high lands east and south of Hudson Bay over the basins of the Great Lakes and the north-central States as far as the Mississippi Valley. It overlapped a previously glaciated region on the south-west, whose drift was derived from ice which moved southward from the central portion of Canada. The evidence for separating the drift of the Illinois glacial lobe from the outlying and underlying drift is briefly stated. Remarkable instances of the transportation of limestone ledges are noted. These ledges in some instances occupy an area of several acres. They have been moved westward from the crest of rock ridges without completely destroying their stratification. Descriptions are given of well-defined soils and weathered zones which occur between successive accumulations of drift; various moraines and associated sheets of till are described, and there is a general discussion on the influence of the drift on drainage systems. The thickness of the Illinois drift is estimated at from 100 to 130 feet, and its bearing on water-supply is fully considered. Reference is made to gas-wells. In some the gas appears to be derived from the decay of vegetable matter in the drift; in most cases, however, it is probable that the underlying rocks contribute the gas, which is pent up beneath compact drift beds. A final chapter treats of soils, and these are classified into residuary soils, boulder-clay soils, gravelly, sandy and bluff-loess soils, silts slowly pervious to water, fine silts nearly impervious, and peaty or organic soils. The residuary soils show variations which correspond in a rude way with variations in the structure of the rocks, whether shale, limestone or sandstone, from which they are derived.

ON THE RELATIONS OF RADIATION TO TEMPERATURE.[1]

THE key to this subject is the principle, arrived at independently by Balfour Stewart and Kirchhoff about the year 1857, that the constitution and intensity of the steady radiation in an enclosure is determined by the temperature of the surrounding bodies, and involves no other element. It was pointed out by Stewart[2] that if the enclosure contains a radiating and absorbing body which is put in motion, the temperature being uniform throughout, then the constitutions of the radiation in front of it and behind it will differ on account of the Doppler effect, so that there will be a chance of gaining mechanical work in the restoration of a uniform state. There must thus be some kind of thermodynamic compensation, which might arise from æthereal friction, or from work required to

[1] A paper read by Dr. J. Larmor, F.R.S., before Section A of the British Association at Bradford, September, 1900.

[2] *Brit. Assoc. Report*, 1871; *cf.* also *Encyc. Brit.*, art. "Radiation" (1886), by Tait.

produce the motion of the body against pressure excited by the surrounding radiation. The hypothesis of friction is now out of court in ultimate molecular physics; while the thermodynamic bearing of a pressure produced by radiation has been developed by Bartoli and Boltzmann (1884), and that of the Doppler effect by Wien (1893).

Application of the Doppler Principle.—The procedure of Wien amounts to isolating a region of radiation within a perfectly reflecting enclosure, and estimating the average shortening of the constituent wave-lengths produced by a very slow shrinkage of its volume. The argument is, however, much simplified if the enclosure is taken to be spherical and to remain so; for it may then be easily shown that each individual undulation is shortened in the same ratio as is the radius of the enclosure, so that the undulatory content remains similar to itself, with uniformly shortened wave-lengths, whether it is uniformly distributed as regards direction or not, and whatever its constitution may be. But if there is a very small piece of a material radiator in the enclosure, the radiation initially inside will have been reduced by its radiating and absorbing action to that corresponding to its temperature. In that case the shrinkage must retain it always, at each stage of its transformation, in the constitution corresponding to some temperature. Otherwise differences of temperature would be effectively established between the various constituents of the radiation in the enclosure; these could be permanent in the absence of material bodies; but if the latter are present this would involve degradation of their energy, for which there is here no room, because, on the principles of Stewart and Kirchhoff, the state corresponding to given energy and volume and temperature is determinate. Thus we infer that if the wave-lengths of the steady radiation corresponding to any one temperature are all altered in the same ratio, we obtain a distribution which corresponds to some other temperature in every respect except absolute intensities.

Direct Transformation of Mechanical Energy into Radiation.[1]—There is one point, however, that rewards examination. When undulations of any kind are reflected from an advancing wall, there is slightly more energy in the reflected beam than there was in the incident beam, although its length is shorter on account of the Doppler effect. This requires that the undulations must oppose a resistance to the advancing wall, and that the mechanical work required to push on the wall is directly transformed into undulatory energy. In fact, let us consider the mechanism of the reflexion. Suppose the displacement in a directly incident wave-train, with velocity of propagation c, to be $\xi = a\cos(mx - mct)$; that in the reflected train will be $\xi' = a'\cos(m'x + m'ct)$, where a', m' are determined by the condition that the total displacement is annulled at the advancing reflector, because no disturbance penetrates beyond it; therefore, when $x = vt$, where v is its velocity, $\xi + \xi' = 0$. Thus we must have $a' = -a$, and $m' = m\dfrac{c-v}{c+v}$, in agreement with the usual statement of the Doppler effect when v is small compared with c. Observe, in fact, that the direct and reflected wave-trains have a system of nodes which travel with velocity v, and that the moving reflector coincides with one of them. Now the velocities $d\xi/dt$ and $d\xi'/dt$ in these two trains are not equal. Their mean squares, on which the kinetic energy per unit length depends, are as m^2 to m'^2. The potential energies per unit length depend on the means of $(d\xi/dx)^2$ and $(d\xi'/dx)^2$, and are of course in the same ratio. Thus the energies per unit length in the direct and reflected trains are as m^2 to m'^2, while the lengths of the trains are as m' to m; hence their total energies are as m to m'; in other words, the reflected train has received an accretion of energy equal to $1 - m'/m$ of the incident energy, which can only have come from mechanical work spent in pushing on the reflector with its velocity v. The opposing pressure is thus in numerical magnitude the fraction $\left(1 - \dfrac{m'}{m}\right)\dfrac{c}{v}$ of the density of the incident energy, which works out to be $\dfrac{c^2 - v^2}{c^2 + v^2}$ of the intensity of the total undulatory energy, direct and reflected, that is in front of the reflector.

[1] The present form of this argument arose out of some remarks contributed by Prof. FitzGerald, and by Mr. Alfred Walker of Bradford, to the discussion on this paper. Mr. Walker points out that by reflecting the radiation from a hot body, situated at the centre of a wheel, by a ring of oblique vanes around its circumference, and then reversing its path by direct reflexion from a ring of fixed vanes outside the wheel, so as to return it into the source, its pressure may be (theoretically) utilised to drive the wheel, and in time to get up a high speed if there is no load: the thermodynamic compensation in this very interesting arrangement lies in the lowering of the temperature of the part of the incident radiation that is not thus utilised.

When v is small compared with c, this agrees with Maxwell's law for the pressure of radiation. This case is also theoretically interesting, because in the application to æther-waves ξ is the displacement of the æther elements whose velocity $d\xi/dt$ represents the magnetic force; so that here we have an actual case in which this vector ξ, hitherto introduced only in the theoretical dynamics of electron-theory, is essential to a bare statement of the facts. Another remark here arises. It has been held that a beam of light is an irreversible agent, because the radiant pressure at the front of the beam has nothing to work against, and its work is therefore degraded. But suppose it had a reflector moving with its own velocity c to work against; our result shows that the pressure vanishes and no work is done. Thus that objection to the thermodynamic treatment of a single ray is not well founded.

This generalisation of the theory of radiant pressure to all kinds of undulatory motion is based on the conservation of the energy. It remains to consider the mechanical origin of the pressure. In the special case of an unlimited stretched cord carrying transverse waves the advancing reflector may be a lamina, through a small hole in which the cord passes without friction: the cord is straight on one side of the lamina, and inclined on the other side on account of the vibration; and it is easily shown that the resultant of the tensions on the two sides provides a force acting on the lamina which, when averaged, agrees with the general formula. In the case of an extended medium with advancing transverse waves, which are reflected directly, the origin of the pressure is not so obvious, because there is not an obvious mechanism for a reflector which would sweep the waves in front of it as it advances. In the æthereal case we can, however, on the basis of electron-theory, imagine a constitution for a reflector which will turn back the radiation on the same principle as a metallic mirror totally reflects Hertzian waves, and thus obtain an idea of how the force acts.

The case of direct incidence has here been treated for simplicity; that of oblique incidence easily follows; the expression for the pressure is reduced in the ratio of the square of the cosine of the angle of incidence. If we average up, after Boltzmann, for the natural radiation in an enclosure, which is incident equally at all angles, we find that the pressure exerted is one-third of the total density of radiant energy.

Adiabatics of an enclosed Mass of Radiation, and resulting General Laws.—Now consider an enclosure of volume V containing radiant energy travelling indifferently in all directions, and of total density E; and let its volume be shrunk by δV. This requires mechanical work $\frac{1}{3}E\delta V$, which is changed into radiant energy: thus

$$EV + \tfrac{1}{3}E\delta V = (E - \delta E)(V - \delta V),$$

where $E - \delta E$ is the new density at volume $V - \delta V$. This gives $\frac{4}{3}E\delta V = V\delta E$, or $E \propto V^{-\frac{4}{3}}$.

As already explained, if the original state has the constitution as regards wave-lengths corresponding to a temperature T, the new state must correspond to some other temperature $T - \delta T$. Thus we can gain work by absorbing the radiation into the working substance of a thermal engine at the one temperature, and extracting it at the other; as the process is reversible, we have by Carnot's principle

$$\tfrac{1}{3}E\delta V/EV = -\delta T/T,$$

so that $T \propto V^{-\frac{1}{3}}$.

Thus $E \propto T^4$, which is Stefan's law for the relation of the aggregate natural radiation to the temperature, established theoretically on these lines by Boltzmann.

Moreover, the Doppler principle has shown us that in the uniform shrinkage of a spherical enclosure the wave-lengths diminish as the linear dimensions, and therefore as $V^{\frac{1}{3}}$, or inversely as T by the above result. Thus in the radiations at different temperatures, if the scale of wave-length is reduced inversely as the temperature the curves of constitution of the radiation become homologous, *i.e.*, their ordinates are all in the same ratio. This is Wien's theoretical law.

These relations show that the energy of the radiation corresponding to the temperature T, which lies between wave-lengths λ and $\lambda + \delta\lambda$, is of the form $\lambda^{-5}f(\lambda T)\delta\lambda$. The investigation, theoretical (Wien, Planck, Rayleigh, &c.) and experimental

(Lummer and Pringsheim, Paschen, &c.), of the form of this function f is perhaps the most fundamental and interesting problem now outstanding in the general theory of the relation of radiation to temperature. The theoretical relations on which this expression is founded have been shown to be in agreement with fact; and it appears that the form $c_1 e^{-c_2/\lambda T}$ fairly represents $f(\lambda T)$ over a wide range of temperature.[1] These relations have been derived, as usual, from a dynamical discussion of the aggregate intensity of radiation belonging to the temperature; it may be shown that the same results, but nothing in addition, will be gained by applying the same principles to each constituent of range $\delta\lambda$ by itself, assigning to each its own temperature.

SCIENTIFIC SERIALS.

American Journal of Mathematics, vol. xxii., No. 4.—Asymptotic evaluation of certain totient sums, by D. N. Lehmer, is an attempt to account for what seems to be a remarkable law, first observed in particular cases in 1895. It is thus stated: Consider any set s of k linear forms, $an+b_i (i=1 \ldots k)$, all of which have the same modulus a, and where $[a, b_i]=1$, $[a, b]$ stands for the G.C. divisor of a and b. Consider, further, a function $\theta_s(x)$ such that $\theta_s(x)=1$ or 0, according as each of the prime divisors of x belongs to one of the forms of the set s or not. If, then, $\nu(x)$ denotes the number of distinct primes in x, we have

$$\lim_{N=\infty} \frac{\sum_{x=1}^{N} 2^{\nu(x)} \theta_s(x)}{N} = \text{constant}$$

The author's aim here is to prove this law where s is the set of linear forms belonging to a binary quadratic form. He also determines the constant in this case (pp. 293-335). Dr. E. H. Moore's paper concerning Klein's group of $(n+1)!$ ary collineations is a modification of a communication made to the American Mathematical Society, December 30, 1898. This short note is related to several papers by the author, amongst others with one communicated to the London Mathematical Society (December 1896, *see* vol. xxviii., p. 357). The closing paper (pp. 343-380), is one by H. E. Slaught, entitled "The Cross-ratio Group of 120 Quadratic Cremona Transformations of the Plane. Part i., Geometric representation." The group specially studied is a particular case, $n=5$ of the general cross-ratio group of order $n!$ Its consideration was undertaken as a dissertation at the University of Chicago at the suggestion and under the direction of Dr. Moore, and so is closely connected with the two papers referred to above. A few plates of figures are given at the end of the number, and also an obituary notice of Prof. T. Craig by Prof. S. Newcomb.

Symons's Monthly Meteorological Magazine, December.—Climatological table for the British Empire for the year 1899. This interesting summary of the climatological tables which have been published for a number of selected stations for the last quarter of a century shows that, generally speaking, the extreme values fall much to the same stations as usual. Adelaide records the maximum shade temperature, 113°·6 (in February). This temperature has only once been exceeded in these tables—viz. by 114°·2 in 1876. It had also the maximum temperature in the sun, 175°·7; this value is also unusual, the only higher reading being 180°, in 1882. It is also the driest station, the mean humidity being 59. The dampest station was Colombo, mean humidity 79; this station also records the highest mean temperature, 81°·9, and the greatest rainfall, 73·5 inches. The lowest temperature in the shade was −46°·5 at Winnipeg (in February); this station had also the greatest range in the year, 135°·9, the greatest mean daily range, 22°·3, and the lowest mean temperature, 34°·2. Mauritius was the most cloudy station—viz. 5·7; this extreme has several times been recorded at London, but in 1899 the cloud value was 5·6, only three out of the preceding forty years having been less cloudy.

[1] This might, however, be multiplied by $(T\lambda)^k$, and the experiments would hardly discriminate between the values zero (Wien), unity (Rayleigh), and one-half (Thiesen) for k, the latter value, which is entirely empirical, seems to fit best.

SOCIETIES AND ACADEMIES.

LONDON

Royal Society, December 6.—"On the Bacterial Disease of the Turnip (*Brassica napus*)." By M. C. Potter, M.A., F.L.S., Professor of Botany in the University of Durham College of Science, Newcastle-upon-Tyne. Communicated by Sir M. Foster, Sec. R.S.

This paper gives the results of an investigation into the cause of a special disease of the turnip crop. The disease was discernible in plants still growing in the fields, some roots being found which were quite rotten, the decaying portion being white with a highly offensive and peculiar smell. The most careful microscopic search failed to detect any trace of hyphæ of the higher fungi in the decaying mass, but only a swarming mass of bacteria. The tissues were completely disorganised, the cells separating from each other along the middle lamella; the cell-walls were soft, swollen and faintly striated; the protoplasm, too, had lost its natural colour and become slightly brown and contracted. The disease could be readily communicated to sound, growing roots by merely making a slight incision and inoculating the root at the injured surface.

After a long series of cultures a bacterium was isolated, and pure cultures obtained grown from a single individual, which produced all the symptoms of "white rot" when sown upon sterile blocks of living turnip. This bacterium rapidly liquefies gelatine, it is a short, motile rod with a single polar flagellum, and, adopting Migula's classification, has been described under the name of *Pseudomonas destructaus*. When growing in a living plant tissue or in nutrient solutions, a cytase is secreted; this was isolated by the well-known method of precipitation by alcohol, and has been shown to cause the dissolution of the cells and the softening and swelling of the cell-walls of the host.

The appearance of the diseased tissue could not be entirely explained by the action of the cytase. It was found that the boiled, filtered juice of a turnip, which had become rotten through the influence of a pure culture of *P. destructaus*, had a powerful toxic effect upon the living plant cell. This toxin proved to be oxalic acid. A reaction probably takes place between the calcium pectate of the middle lamella and the oxalic acid produced by the bacteria, the calcium pectate neutralising the oxalic acid and thus permitting the continued growth of the bacteria.

The action of this bacterium upon living plant tissues is similar to that of some of the parasitic fungi; in both cases the invading organism produces oxalic acid, which acts as a toxin to the protoplasm and, decomposing the calcium pectate, furthers the dissolution of the cells; and also there is the secretion of a cytase, which has a destructive action upon the cell-wall and intercellular substance. The question of the parasitism of the bacteria thus stands in these respects on the same platform as that of the fungi, and a complete homology is established between them.

From numerous observations in the fields it would appear that *P. destructaus* is always introduced at a wounded surface, and through the agency of slugs and larvæ.

"On the Tempering of Iron Hardened by Overstrain." By James Muir, B.Sc., B.A., Trinity College, Cambridge, 1851 Exhibition Research Scholar, Glasgow University. Communicated by Prof. Ewing, F.R.S.

It is well known that iron hardened by overstrain—for example, by permanent stretching—may have its original properties restored again by annealing—that is, by heating it above a definite high temperature and allowing it to cool slowly. Experiments described in the paper, of which this is an abstract, show, however, that if iron hardened by overstrain be raised to any temperature above 300° C., it may be partially softened in a manner analogous to the ordinary tempering or "letting down" of steel which has been hardened by quenching from a red heat. This tempering from a condition of hardness induced by overstrain, unlike ordinary tempering, is applicable not only to steel, but also to wrought iron, and possibly to other materials which can be hardened by overstrain and softened by annealing.

The experiments described in the paper were all carried out on rods of iron and steel about ⅜ths of an inch in diameter and 11 inches long, the elastic condition of the material being in all cases determined by means of tension tests, in which the hardness of the material was indicated by the position of the yield-point.